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AN ANALYSIS ON SHORTEST PARALLEL QUEUEING SYSTEM WITH JOCKEYING

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ABSTRACT

In this paper, we are investigating shortest parallel queues with jockeying. In general the parallel queue system and customer join the fastest one. The capacity of each queue is restricted to N including the one being served. There is a FIFO service discipline in which the input stream is Poisson having rate λ. The service time of any customer at server ` i' (i=1, 2) is exponential with parameter μ_i. The state probability and loss probability of this model are obtained. The performance measures are obtained and optimized. On arrival a job joins the shortest queue and in case both queues have equal length. To obtain mean number of waiting in the system and customer

KEYWORDS: Steady State Solution, Shortest Queue, Jockeying, SINGLE Server

INTRODUCTION

Queuing theory is a art of Mathematics that studies different aspect of act of waiting in lines. This research articles will take a brief discussion of the customer should begin to understand the basic ideas of how to determine useful information such as average waiting times from particular Queuing system.

The first paper on Queuing theory 'The theory of probabilities and telephone conversations', was published in 1909 by A.K.E rlank, now considered the father of the field. His work with the Copenhagen telephone company is what prompted his initial foray into the field. Erlang switch board problem laid the path for the modern Queuing theory.

The concept of jockeying is one of the important customer strategies. If refers to the movements of customers who have the option of switching from one queue to another, where parallel servers, each having a separate and distinct queue are available. Most of us have been jockeying for years-switching lines in auto license offices and supermarkets, changing lanes on high ways, changing routes in rush-hour traffic and changing suppliers (server) when confronted by long queues. At other times (changing routes), we jockey without information on the state of new line. In some cases, we join a preferred line and do not jockey until we have suffered some delay. Haight [5] introduces the shortest parallel queue model. Kingman [6], Flatto and McKean [4] show that the functional equations for the bivariate generating function are analyzed by using the uniformisation of a polynomial of 2 variables. Fayolle and Iasnogorodski [3], Choen and Boxma [1] show how the analysis can be reduced to be the solution of a Riemann-Hilbert boundary value problem. However the apporach does not lead to explicit expression for the equilibrium probabilities. The only strategy studied in any detail is that of impatient customer, who leaves the waiting system [1, 2, and 3].

Aden et.al showed for the symmetric shortest queue problem that the steady state distribution of the queue lengths of the two queues can be found in an elementary way directly from the equilibrium equations. Conolly [2] discussed the finite waiting room version of the shortest queue problem and showed that this problem can be solved efficiently, essentially by dimension reduction. Knessl [18] considered two parallel M/M/∞ queues. They assumed that the arrival rate is λ is large compared to the two service rates μ_1, μ_2 .

THE BASIC MODEL

To begin understanding queues, we must first have some basic knowledge of probability theory. We will review the exponential and Poisson probability distributions.

The exponential distribution with parameter λ is given that $\lambda e^{-\lambda t}$ for t≥ 0. If T is a continuous random variable that represents interracial with exponential distribution, then

P (T$\leq t$) =1-$e^{-\lambda t}$ and

P (T$\geq t$) = $e^{-\lambda t}$

It is also useful to note the exponential distributions relation to the Poisson distribution. The Poisson distribution is used to determine the probability of a certain number of arrivals occurring in a given time period. The Poisson distribution with parameter λ is given by

$\frac{(\lambda t)^n e^{-\lambda t}}{n!}$ Where

N is the number of arrivals we find that if we set n=0, the Poisson distribution gives us

$e^{-\lambda t}$

Which is equal to P (T>t) from the exponential distribution. The relation here also makes sense. Then the above description deals with how the Poisson distribution converts to interarrival of exponential distribution. The interarrival time here, of course, is the time between customer arrivals and thus is a period of time with zero arrivals.

MODEL DESCRIPTION

- In the two queues in parallel with joking model, the arrival process of customers is a Poisson process with arrival rate λ .
- The Queuing system consists of two parallel servers with a different rate μ_1 and μ_2 respectively. The capacity of each queue is restricted to L including one being served.
- Service is first in first out served in any queue (but a jockeying customer joins and the end of the new line).

Some of the customers can join any of the queues, or choose to wait and join at some time later. Assume that the parallel queue length are initially equal to K, for some integer K>0. Thus should be a fairly common situation since if one of the queues is shorter than the other queue arriving customers will join the shorter queue and to equalize them. Suppose a pair of customers at fast food restaurant each join a different queue. When one of the customer reaches the server, that the server serve fastly compare for the another server then another customer to skip and join the fastly served queue We are interested i the system time for a pair of arriving customers under variety strategies,

- Customer enters different lines immediately.
- Customer makes a jockeying who fast server the system.

Result (i)

Let X_1 and X_2 be independent random variables. Let Z=min { X_1, X_2 } then

E(z)=P($X_1 < X_2$)E(X_1 / $X_2 < X_2$)+P($X_2 < X_1$)E(X_2 / $X_2 < X_1$).

Corollary

For k be an integer $\int_{\mu}^{\infty} \frac{1}{\Gamma(k)} z^{k-1} e^{-z} dz = \sum_{x=0}^{k-1} \frac{\mu^x e^{-\mu}}{x!}$

This result relates the probability that one gamma variable exceeds another to the fail probabilities of a negative binomial random variable.

Analysis of the Strategy

Assume that the first customer join queue 1 immediately. If it turns out queue1 has a very slow server relative to another queues. The problem becomes a two queue problem. It was begin with n customers, and then waiting before joining can be advantages if the difference in the service rates of the two queues is sufficiently large. Considered two parallel M/M/ systems of queues with arrival rate λ and generates $\mu_1 = \mu_2 = \mu$. In this result for the equilibrium or steady state condition and do not take into account the dynamic of jockeying. Thus we have omitted, in this paper, any consideration of detailed jockeying history of specific customer, nor can we calculate the delay suffered by a customer who jockeys exactly n times before being served. The steady state equations do allow the calculation of the mean line length and its variance, from which the average delay can be calculated.

The merit of service to the customer used is the average waiting line length. In the absence of jockeying the delay suffered by a customer is proportional t o the line length if the service disciplines are FCFS. For other disciplines the mean waiting time is the same as for the first-come first- served discipline, but the variance of waiting time is greater. Some of the time server becomes idle.

Performance Measure

In this system each server has a unique waiting line to service. Once a customer has selected a line, he remains in the line. The customer's strategy is to join the shorter line.

Customer Strategy

A unit arrives to find

- Both servers engage: it joins the shorter line. If both lines are of equal length, it chooses which is fastly moving either channel with equal probability.
- Only one server is free; it occupies the free channel.
- Both servers free; it chooses either channel with equal probability.

A modification of the strategy can be called, "Tellers window performance". The customer strategy is changed, so that when the customer has a choice (either both servers free or birth waiting lines of the same length), he selects server with probability π_1 and server 2 with probability π_2, where $\pi_1 + \pi_2 = 1$.

Following the queue discipline and customers strategies, we have the transition equations:

$$\frac{d}{dt}Q_{00} = -\lambda Q_{00} + -\lambda_1 Q_{10} + \mu_2 Q_{01} \tag{1}$$

$$\frac{d}{dt}Q_{10} = -(\lambda + \mu_1)Q_{10} + \frac{\lambda}{2}Q_{00} + \mu_1 Q_{20} + \mu_2 Q_{11} \tag{2}$$

$$\frac{d}{dt}Q_{01} = -(\lambda + \mu_2)Q_{01} + \frac{\lambda}{2}Q_{00} + \mu_1 Q_{11} + \mu_2 Q_{02} \tag{3}$$

$$\frac{d}{dt}Q_{n_1,n_2} = -(\lambda + \mu_1 + \mu_2)Q_{n_1,n_2} + \Delta_1 Q_{n_{1-1},n_2} + \Delta_2 Q_{n_1,n_2} + \mu_1 Q_{n_{1+1},n_2} + \mu_2 Q_{n_1,n_2+1}(n_1, n_2 \geq 1) \tag{4}$$

Where

$$\Delta_1 = \begin{cases} 0; n_1 - 1 > n_2 \\ \lambda/2; n_1 - 1 = n_2 \\ \lambda : n_1 - 1 < n_2 \end{cases} \tag{5}$$

$$\Delta_2 = \begin{cases} 0; n_2 - 1 > n_1 \\ \lambda/2; n_2 - 1 = n_1 \\ \lambda : n_2 - 1 < n_1 \end{cases} \tag{6}$$

Writing the equations for Q_{nr} for all n and fixed r, multiply the n^{th} equation by ξ^n, and writing the generating function $g_r(\xi)$ as

$$g_r(\xi) = \sum_{n=0}^{\infty} \xi^n Q_{nr} \tag{7}$$

We obtain the following equations:

$$\frac{d}{dt}\sum \xi^n Q_{n0} = g_0(\xi)[\frac{\mu_1}{\xi} - \mu_1 - \lambda] + g_0(0)[\mu_1 - \frac{\mu_1}{\xi}] + \mu_2 g_1(\xi) + \frac{\lambda}{2}\xi Q_{00}, \tag{8}$$

$$\frac{d}{dt}\sum \xi^n Q_{n1} = g_1(\xi)[\frac{\mu_1}{\xi} - \mu_1 - \lambda - \mu_2] + g_1(0)[\mu_1 - \frac{\mu_1}{\xi}] + \mu_2 g_2(\xi) + \lambda g_0(\xi) - \frac{\lambda}{2}Q_{00} + \lambda\xi Q_{01} + \frac{\lambda}{2}\xi^2 Q_{11}, \tag{9}$$

$$\frac{d}{dt}\sum \xi^n Q_{n2} =$$
$$g_2(\xi)[\frac{\mu_1}{\xi}-\mu_1-\lambda-\mu_2]+g_2(0)[\mu_1-\frac{\mu_1}{\xi}]+\mu_2 g_3(\xi)+\lambda g_1(\xi)-\lambda Q_{01}-\frac{\lambda\xi}{2}Q_{11}+\lambda\xi Q_{02}+\frac{\lambda}{2}\xi^2 Q_{12}+\frac{\lambda}{2}\xi^2 Q_{22}, \tag{10}$$

$$\frac{d}{dt}\sum \xi^n Q_{n3} =$$
$$g_3(\xi)[\frac{\mu_1}{\xi}-\mu_1-\lambda-\mu_2]+g_3(0)[\mu_1-\frac{\mu_1}{\xi}]+\mu_2 g_4(\xi)+ \tag{11}$$
$$\lambda g_2(\xi)-\lambda Q_{02}-\lambda\xi Q_{12}-\frac{\lambda\xi^2}{2}Q_{22}+\lambda\xi Q_{03}+\lambda\xi^2 Q_{13}+\lambda\xi^3 Q_{23}+\frac{\lambda}{2}\xi^4 Q_{33},$$

Now, multiply the r^{th} equation of equation (9) by ξ^r and summing over all values of r, we have

Since we are only interested in the steady state solutions, the left hand side is zero. After reorganizing terms, we have

$$0=[(\mu_1+\mu_2)/\xi-\lambda-\mu_1-\mu_2+\lambda\xi]\sum_{r=1}^{\infty}\xi^r g_r(\xi)+[((\mu_1+\mu_2)/\xi)]\sum_{r=0}^{\infty}\xi^r g_r(0)+[(\mu_1+\mu_2)/\xi]g_0(\xi) \tag{12}$$

We let $g_r(\xi)=a_r(1+\rho\xi+(\rho\xi)^2+(\rho\xi)^3+.........)= a_r \big/ (1-\rho\xi)$ (13)

Where $\rho = \lambda \big/ (\mu_1+\mu_2)$

Substituting equation (14) in equation (13), we find

$a_r = \rho^r a$

And $g_r(\xi)=\rho^r a_0/(1-\rho\xi)$

Now, we know that $g_r(1)=\sum_{r=0}^{\infty} Q_{nr} = P_r^2$

And there for

$$\sum_{r=0}^{\infty} g_r(1)=\sum_{r=0}^{\infty} P_r^2 = 1$$

Thus, we have

$a_0=(1-\rho)^2$ And $g_r(1)=P_r^2=\rho^r(1-\rho)$, $g_0(1)=P_0^2=(1-\rho)=P_0^1$

Similarly,

$g_r(0)=Q_{0r}=\rho^r a_0+\rho^r(1-\rho)^2$, And $g_0(0) = Q_{00} = (1-\rho)^2$.

By suitable manipulation, we find the other measures of interest

$$\bar{n}_1 = \bar{n}_1 = \bar{n}_2 = \rho a_0 /(1-\rho)^3 = \rho \big/ (1-\rho) \tag{14}$$

=mean number at queue i,

$$\bar{w}_1 = \bar{w}_1 = \bar{w}_2 = \bar{n}_1 - \rho = \rho^2 /(1-\rho) \tag{15}$$

=mean length of waiting line i,

$$r_1 = (\mu_1 \big/ \lambda)(1-P_0^1) = \mu_1 \rho \big/ \lambda \tag{16}$$

=fraction of customer served in queue 1,

$$r_2 = (\mu_2 \big/ \lambda)(1-P_0^2) = \mu_2 \rho \big/ \lambda \tag{17}$$

=fraction of customers served in queue2.

For the more general case, 'Tellers' windows with preferences', $\lambda \big/ 2$ in equation (2) is replaced by $\lambda \pi_1$, and in carrying Out the summation to obtain equation (13),these terms again disappear. Thus, the general case yields the same results as the case in which $\pi_1 = \pi_2 = \frac{1}{2}$; customers preference do not affect the standards of service in queue situations which follow the general rule, 'Choose shortest line and staying it'. In both case, the system yields a steady state condition in which the mean waiting lines have the same average independent of the differences in the service rates. The mean waiting time in a line, however, does depend on the service rate the line. If the service rates are vastly different, the customer who arrives at a time in which the slower server has a shorter line suffers much longer delays than one who finds the other line free. Further, the proportion of customers served by the line i is

$$(\mu_1 \big/ \mu_1 + \mu_2) .$$

Numerical Results

The computational results obtained by employing the exhaustive numerical analysis technique are discussed through tables and graphs. For different values of parameters λ, μ_1, μ_2

We carried out a set of experiments measuring to study the effect of traffic intensity $\rho = \lambda \big/ \mu_1 + \mu_2$

On Steady State Probability

Consider the following values apply in above formula, we get

$$\lambda = 0.30, \mu_1 = 0.80 \quad and \quad \mu_2 = 1.0$$

Table 1

Iteration Number	Exact Results		Optimal Results	
	E(T)	E(N)	E(T)	E(N)
100	2.7692	0.8308	2.222	0.6667
1000	2.7692	0.8308	2.222	0.6667
5000	2.7692	0.8308	2.222	0.6667

Using the above data we get the variation of mean number of the queue (E(T)) and mean length of waiting time in line(E(N)),

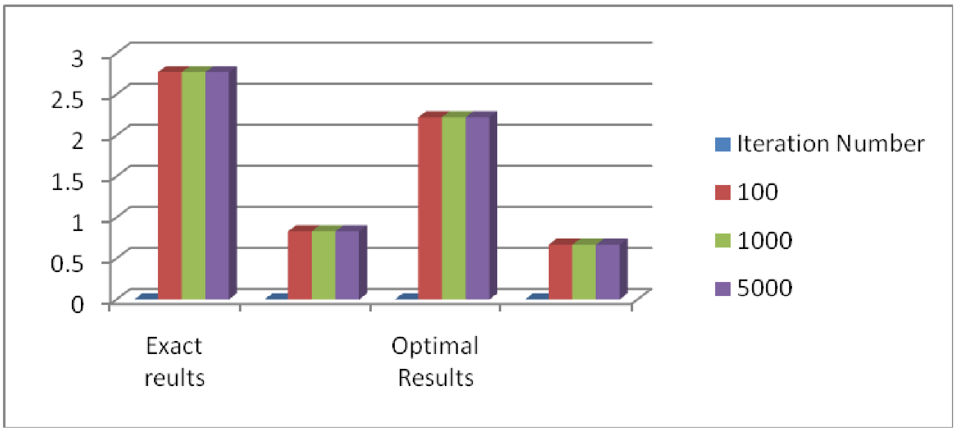

Figure 1

CONCLUSIONS

In this research article, we have presented to shortest parallel queuing system with jockeying and a performance analysis for two parallel queues with jockeying and restricted capacities. In this model, steady state equations, the mean number of customers and loss probability are obtained. The driving force in the system, in all of its forms, is the instantaneous jockeying priciple.The application of Queueing theory extends well beyond waiting in line at a bank. It may take some creative thinging, but if there is any sort of scenario where time passes before a particular event occurs, there is probably some way to develop it into a Queueing model.

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